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The noise in the 35-d cycle of Her X-1

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ABSTRACT

We calculate the power density spectrum of fluctuations in the X-ray turn-on times of Her X-1, including new data which extend the X-ray and optical flux observations to a span of over 20 yr with 221 distinct 35-d high–low flux cycles. If we assume that turn-on times define the period of the 35-d clock, this statistical interpretation of turn-on behaviour is consistent with a white-noise process in the first derivative of the 35-d phase fluctuations (or a random walk in clock phase). We discuss the implications of considering the 35-d clock mechanism as a noise process.

Key words: methods: statistical – binaries: close – stars: individual: Her X-1 – X-rays: stars.

1 INTRODUCTION

In addition to an 808-mHz neutron star rotational frequency and a 1.7-d orbital period, the Her X-1/HZ Her system also shows a third periodicity, a 35-d high–low X-ray flux cycle (Tananbaum et al. 1972). The 35-d flux cycle is marked by alternating main-high and short-high states of 11- and 5-d duration respectively. These states are separated by intervals of relatively low X-ray flux (Jones & Forman 1976). The short-high state is now accepted as a regular feature of the 35-d high–low flux cycle (Deeter et al. 1993). Various models have been proposed to explain the observed character of the 35-d high–low cycle (Brecher 1972; Boynton et al. 1973; Pines, Pethick & Lamb 1973; Roberts 1974; Lamb et al. 1975; Gerend & Boynton 1976; Petterson 1975, 1977; Mazeh & Shaham 1977; Crosa & Boynton 1980; Meyer & Meyer-Hofmeister 1984; Trümper et al. 1986). However, there is still no adequate theoretical explanation for this phenomenon. Most of the phenomenological models assume that a disc periodically occults the X-ray source. Basically, four kinds of model have been proposed to describe the 35-d clock mechanism: (i) forced precession of the companion star HZ Her, leading to an 'enslaved' accretion disc (Katz 1973; Roberts 1974; Petterson 1975); (ii) a tilted, precessing disc, proposed by Petterson (1975, 1977); (iii) a tilted accretion disc undergoing 'apparent' counter-precession opposite to the sense of orbital motion (Gerend & Boynton 1976; Crosa & Boynton 1980), and (iv) neutron star free precession (Pines et al. 1973; Lamb et al. 1975; Trümper et al. 1986). Recent studies of the orbital solution of the Her X-1/HZ Her system (Deeter et al. 1991) and the evolution of pulse profiles in the 35-d cycle (Deeter et al. 1993) indicate that the most favourable model is the tilted, counter-precessing accretion disc; moreover, this model is supported by detailed aspects of both X-ray and optical observations.

The *stability* of the 35-d clock is also an important aspect of an appropriate model. Recent studies concerning the timing noise in the high–low flux cycles, as characterized by the time of X-ray turn-on, have been carried out by Boynton, Crosa & Deeter (1980), Staubert, Bezler & Kendziorra (1983) and Ögelman (1987), where data spanning about 70, 100 and 142 cycle respectively were used. Boynton et al. (1980) showed that a close relationship exists between the timing of turn-ons, the X-ray dips and the optically inferred precessional motion of the accretion disc. They argued that the origin of the 35-d clock cannot lie in the forced precession of HZ Her, nor in the free precession of Her X-1. They concluded that a primary clue to an appropriate model is that the fluctuations in turn-on time are a random walk and therefore correspond to white noise in the turn-on rate (frequency). Ögelman (1987) modelled the main-high-state turn-on data using autoregressive time-series methods, and found that the clock period has an intrinsic rms variance which lies between 0.63 and 1.35 d^2 $cycle^{-1}$, consistent with the strength of the random walk in turn-on time calculated by Boynton et al. (1980).

In this paper, we present a power density spectrum of fluctuations in the first derivative of the 35-d cycle phase fluctuations (turn-on times), calculated from data extending over 221 consecutive 35-d cycles. In Section 2, we describe the turn-on data and construct the power density spectrum using the mean-squared residuals technique developed by Deeter (1984).

2 DATA BASE AND CONSTRUCTION OF THE POWER SPECTRUM

In this analysis we use estimated times of X-ray turn-on for Her X-1 taken from published literature. Most of the times of turn-on are obtained directly from X-ray light curves (Crosa & Boynton 1980), but some are indirectly inferred from systematic 35-d variations in the *optical* light curve (e.g. Boynton et al. 1980; Thomas et al. 1983). The resulting data set consists of 59 turn-ons (Table 1; see References section for full reference details). In Fig. 1 we plot the phase residuals of these turn-ons, $\Delta\phi_N = (T_N - NP_{35})/P_{35} - \langle (T_N - NP_{35})/P_{35} \rangle$, where T_N is the estimated time of turn-on for the Nth cycle and $P_{35} \sim 34.9$ d is the average period of the high–low cycle deduced by Crosa & Boynton (1980) and Ögelman (1987).

Table 1. Observed X-ray and optical turn-on data.

Cycle	Turn-on(JD)	Error(days)	Ref.
-15	797.800	0.800	BOY80
-10	972.700	0.500	CRO80
-7	1077.100	0.300	BOY80
-2	1254.600	0.400	GIA73
-1	1290.024	0.022	CRO80
0	1325.680	0.060	CRO80
1	1361.370	0.040	CRO80
2	1397.000	0.040	CRO80
3	1432.092	0.070	CRO80
4	1466.860	0.050	CRO80
5	1501.649	0.038	CRO80
6	1536.490	0.040	CRO80
7	1572.108	0.023	CRO80
8	1606.090	0.040	CRO80
9	1640.224	0.033	CRO80
10	1675.004	0.030	CRO80
11	1709.190	0.070	CRO80
12	1743.220	0.040	CRO80
13	1777.120	0.500	THO83
14	1812.810	0.500	THO83
15	1848.510	0.500	THO83
16	1884.230	0.500	THO83
17	1918.591	0.410	STA83
18	1953.940	0.500	THO83
24	2163.744	0.017	CRO80
26	2232.800	0.300	BOY80
30	2375.602	0.100	STA83
31	2410.443	0.100	STA83
32	2446.409	0.100	STA83
33	2480.140	0.070	CRO80
34	2514.100	0.020	CRO80
36	2582.800	0.200	BOY80
38	2652.685	0.035	CRO80
43	2827.100	0.300	RIC82
46	2932.300	0.200	RIC82
47	2968.010	0.045	JOSS76
53	3178.130	0.042	DAV77
58	3350.590	0.500	THO83
59	3386.200	0.100	STA83
64	3560.600	0.100	GOR81
69	3732.900	0.300	GOR81
70	3769.000	0.500	SOO90
72	3838.400	0.200	RIC82
73	3873.500	0.200	RIC82
77	4015.350	0.500	THO83
84	4261.000	0.200	RIC82
88	4400.469	0.500	THO83
97	4709.012	0.500	THO83
98	4743.910	0.500	THO83
99	4779.500	1.000	RIC82
109	5127.500	1.000	NAG84
110	5163.840	0.500	THO83
119	5472.000	0.500	DEL83
127	5753.000	1.000	OGE85
128	5788.000	0.500	OGE85
140	6208.300	1.000	OGE85
174	7398.800	1.000	DET93
181	7643.100	0.100	DET93
205	8478.300	0.200	DET93

See References section for full reference details.

Our goal is to deduce a random-walk noise strength for the turn-ons from a low-resolution power density spectrum. The intermediate computation of a spectrum provides a check on the validity of the random-walk model. We use a method of spectral density estimation discussed by Deeter & Boynton (1982) and Deeter (1984). This method entails partitioning of the phase residuals into subsets according to a hierarchy of time-scales, and then fitting the data in each subset with a quadratic polynomial. Each fit is associated with a mean-squared residual, which yields an estimate of the power density on the corresponding time-scale. Conversion into a power density requires an appropriate normalization, which is chosen according to the kind of noise being considered (Cordes 1980; Deeter 1984).

For the case of rth-order red noise with strength S_r, the mean-squared residual for the data spanning an interval with length T is proportional to $S_r T^{2r-1}$. An additional factor depends on the degree m of the polynomial removed prior to computing the mean-square residual, which can be supplied by determining the expected mean-square residual for unit-strength red noise ($S_r = 1$) over a unit interval ($T = 1$), either by Monte Carlo methods (Cordes 1980) or by direct mathematical evaluation (Deeter 1984). The expected mean-square residual after removing a polynomial of degree m over an interval of length T is then given by

$$\langle \sigma_R^2(m, T) \rangle = S_r T^{2r-1} \langle \sigma_R^2(m, 1) \rangle_u, \tag{1}$$

where the subscript u indicates that the expectation has been derived for a unit-strength noise process. Previous studies indicated that first-order red noise is probably associated with Her X-1 turn-ons, so we adopt this noise process in computing the normalization coefficient. We compute the normalization for our specific cases of non-uniformly sampled data through Monte Carlo simulations, and find that there is fairly good agreement with the theoretical evaluation for equi-spaced data sampling (Deeter 1984), with our Monte Carlo results being about 14 per cent smaller.

We compute power density estimates at spacings of nearly an octave, as suggested by Deeter (1984). To do this, we take the entire length of the data as the longest time-scale, and then successively halve the intervals to get down to the shortest practical time-scale. This produces a hierarchy of six time-scales, with the shortest spanning six consecutive 35-d cycles. Intervals with insufficient data points are discarded,

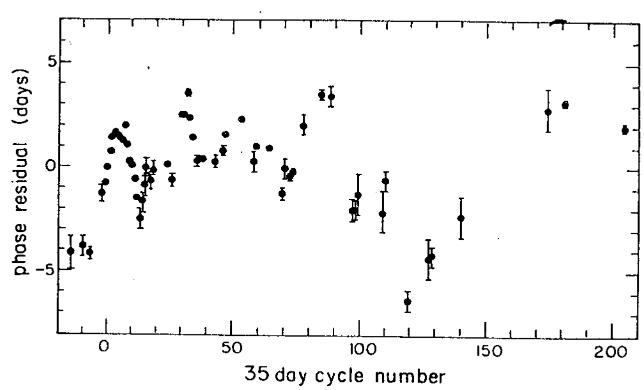

Figure 1. Phase residuals of turn-on data for Her X-1, given in Table 1 (see the text).

and the mean-squared residual for the remaining intervals is calculated and converted into a noise strength using equation (1). We compute the contribution of measurement uncertainty to the noise strength by converting the estimated measurement variances of turn-ons into noise strengths, again using equation (1).

Estimated noise strengths on each time-scale are combined into a single power density estimate by averaging, and a nominal frequency equal to the reciprocal of the time-scale is assigned to this estimate. The effective number of degrees of freedom, ν, for each estimate is determined by generating its statistical distribution through an ensemble of Monte Carlo simulations of a first-order red-noise process, sampled in the same way as our turn-on data. Error bars for the power density error estimates are taken to be the 16 and 84 per cent points on the χ^2_ν distribution. Resulting numerical values for the power density estimates are given in Table 2, and displayed in Fig. 2 in a log–log representation to show as clearly as possible the power-law behaviour. It should be remembered that this spectrum is actually the power density spectrum associated with fluctuations in the first derivative of the 35-d phase, because we have applied a normalization appropriate for a first-order red-noise process.

3 DISCUSSION AND CONCLUSION

The principal goal of this work is to characterize statistically the phase fluctuations in the 35-d cycle as indicated by the turn-on data. We find that the observed power spectrum of the turn-on frequency has a slope $n = -0.0025 \pm 0.17$, and the mean power density (or noise strength) is $2.2 \times 10^{-10}\ \mathrm{s}^{-1}$. This power index is consistent with white noise ($n = 0$) in the derivative of the phase fluctuations. This is equivalent to a random walk in phase fluctuation, which can be characterized by a mean-square phase step-size $\langle \delta\phi^2 \rangle$ and a step-rate R. The resulting cumulative mean-squared deviation in the phase scales with elapsed time t as

$$\langle \Delta\phi^2 \rangle = R\langle \delta\phi^2 \rangle\, t = St, \qquad (2)$$

where $S = R\langle \delta\phi^2 \rangle$ is the noise strength. This random-walk model for phase fluctuations, taken with the observed noise strength, yields an rms variation of $0.9\Delta N^{1/2}$ d between turn-ons separated by ΔN cycles. This rms variation is equivalent to the white-noise variance $\sigma^2 = 0.81\ \mathrm{d}^2\ \mathrm{cycle}^{-1}$, which is consistent with that found by Ögelman (1987) for a shorter data set.

The stability of a clock exhibiting white-frequency noise can be characterized by a quality factor $Q = f/\langle \Delta f^2 \rangle^{1/2}$, where $\langle \Delta f^2 \rangle$ is the rms frequency deviation. For the case at hand, $Q = 39$. What kind of physical mechanisms can lead to such clock noise?

Forced precession of the companion star HZ Her, together with an 'enslaved' accretion disc, has been considered the cause for the 35-d cycle in Her X-1 (Katz 1973; Roberts 1974; Petterson 1975). One of the main indicators of this forced precession would be a slow variation in the observed orbital parameters during the 35-d period (Deeter & Boynton 1976). A recent study of *Ginga* observations indicates that the orbital solutions in the main-high and short-high states agree closely with each other (Deeter et al. 1991). This agreement has yielded strong constraints on the obliquity of the stellar companion, HZ Her, and has strongly ruled against the plausibility of the forced precession of HZ Her and the associated 'enslaved' accretion disc. In terms of clock noise, we also consider this model to be implausible. If the 'enslaved' accretion disc is tightly coupled to the stellar precession, then the observed variability in the 35-d period is too large to be consistent with forced precession. If a loose coupling process is considered then a random walk in phase implies arbitrarily large phase displacements between the precession phases of the disc and star, defying any coupling between them (Boynton et al. 1980).

Free precession of the neutron star has also been considered as providing the underlying clock for the 35-d cycle in Her X-1 (Brecher 1972; Trümper et al. 1986). If the neutron star is rotating around an axis different from one of the principal axes of the body, and the body is a symmetric top, the star will undergo 'free precession' (Bisnovatyi-

Table 2. Power spectrum of noise in the derivative of the 35-d turn-on phase of Her X-1.

log f	log P^a	log P^b	Effective
(1/cycle)	(1/sec)	(1/sec)	d.o.f
-2.34	-11.29	-9.35	3.
-2.04	-10.74	-9.67	6.
-1.74	-10.34	-9.56	10.
-1.44	-10.21	-9.59	15.
-1.14	-10.27	-9.75	19.
-0.84	-10.02	-9.63	16.

Notes: ^{a}contribution of measurement errors; ^{b}power density of the data.

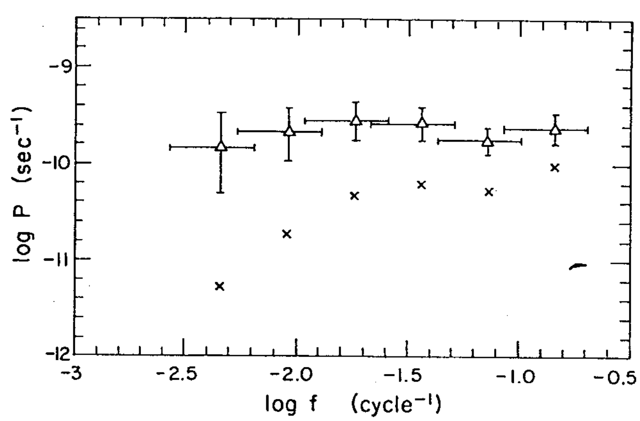

Figure 2. Power density spectrum of the fluctuations in the frequency of the Her X-1 turn-ons, computed by the method described in the text. Power density estimates are indicated by triangles (with error bars), with the contributions from measurement uncertainty shown by the associated crosses (see Table 2). Vertical error bars indicate the stability of each power estimate, being approximate $\pm 1\sigma$ errors on the associated statistical distribution as obtained by Monte Carlo simulations. Horizontal error bars indicate approximate $\pm 1\sigma$ errors of the distribution of analysis frequencies sampled by each estimate, as specified by Deeter (1984).

Kogan, Mersov & Sheffer 1990). Trümper et al. (1986) have suggested that unpinning of a number of strongly pinned vortices with moments of inertia $I_p \sim 10^{-9}$ I(total) can lead to appropriate fluctuations in the 35-d period. Alpar & Ögelman (1987) have shown that coupling of the crustal motion to the crust superfluid would lead to a significant energy dissipation, and concluded that the observed fluctuations of the 35-d period of Her X-1 cannot be due to internal torques associated with pinned vortices. It might be argued that free precession is excited by fluctuations in matter accretion associated with the accretion flow geometry changes at the inner disc edge or in the magnetosphere. Such a mechanism, however, would lead most naturally to white noise in a 35-d phase fluctuation instead of a random walk.

Crosa & Boynton (1980) have suggested that 0.81-d periodic mass-transfer episodes constitute a fourth basic periodicity in the Her X-1/HZ Her system. They argue that a tilted, twisted accretion disc shadows the inner Lagrangian point nominally twice each binary orbit, but, because of the retrograde precession of the 35-d period (opposite to the sense of orbital motion) of the disc with a 35-d period, these shadowing events occur once every 0.81 d, not at half the orbital period (0.85 d). Between the shadowing events, matter is stored at the inner Lagrangian point due to radiation pressure from Her X-1. As the shadow of the inner edge of the accretion disc passes over L_1, matter begins to flow and eventually joins the outer boundary of the accretion disc. Because of the ongoing retrograde precessional motion, this material will have somewhat different angular momentum from that for the previous mass-transfer episode, and consequently the outer boundary of the disc will form at a slightly different orientation (the position of the line of nodes). Thus the discrete mass-transfer events lead to a stepwise rotation of the line of nodes of the tilted outer disc. Through viscous coupling, this progressive re-formation of the outer disc edge causes the inner disc edge to keep pace; the disc configuration is fixed, with orientation determined by the outer edge as a boundary value problem (Petterson 1975). Thus the apparent retrograde precessional motion of the disc is thought to follow from the closed-loop dynamics of disc regeneration through a repeated sequence of mass-transfer events regulated by the X-ray shadow cast on L_1, which in turn is only a self-consistent expression of the changing disc orientation. The time delays in communicating the effects on the disc geometry of this mass-flow process, the free-fall time from L_1 to the outer edge of the disc, and the viscous propagation time of angular momentum through the disc determine both the apparent precessional period and the sense of precession. Noise superposed on this mechanism through random (white noise) fluctuations in these delay-time contributions results in a cumulative (random walk) error in the disc orientation, because of the inherently Markoffian character of the mechanism.

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